manner, together with curves of the separate effects of the Solar Parallax and Solar Declination.

A sheet from the Tide-Gauge Cylinder has also been sent, as a specimen of the regularity of the registered Curves in tranquil weather.

II. "On the Heating of a Disk by rapid Rotation *in vacuo*." By Balfour Stewart, M.A., F.R.S., and P. G. Tait, M.A. (Continuation of a Paper read before the Royal Society on June 15, 1865, and published in the 'Proceedings.') Received October 30, 1866.

16. The apparatus and certain preliminary experiments having been described in the previous paper, the authors now proceed to relate what further experiments have been made.

In the preliminary experiments it was conclusively shown (art. 8) that the effect on the pile caused by rotation of the disk was due to radiant heat, and also (art. 9) that this effect was not due to the heating of the rock-salt, which in most of the experiments was placed before the mouth of the cone.

It was also rendered probable that the effect was not due to radiation from heated air, by the two following considerations:—

(1) Because in order that nearly dry air of such a tenuity might give such a radiation it would require to be heated enormously.

(2) Because when the lampblack was removed from the aluminium disk, leaving it a rough metallic surface, the indication afforded by the galvanometer was reduced to about one-fourth of the amount with the blackened disk.

The following observations tend to strengthen this proof:—

(3) The heating effect is the same in hydrogen or in coal-gas as in air, although there is no question that the absorptive, and therefore the radiative power of coal-gas is much greater than that of air. This is shown by the following sets of experiments, which were made with the blackened aluminium disk insulated with ebonite, and with rock-salt in the cone.

No. of set.	No. of observations in each set.	Time at full speed, in seconds.	No. of turns of handle at full speed.	Heat indication.		Nature of gas.	Tension of gas, in inches.	Probable purity of gas (absolutely pure gas =100).
				Divisions.	Fahr.			
VII.	2	30	22	22·5	...	hydrogen	0·6	95
VIII.	3	30	22	23·3	...	air	0·7	
IX.	2	30	20	...	0°·95	hydrogen	0·5	97
X.	2	30	20	...	0°·87	air	1·1	
XI.	3	30	20	...	0°·85	hydrogen	0·25	98·5
XII.	3	30	20	...	0°·86	coal-gas	0·25	95

(4) It may be objected to (2) that the greater heating effect from a

blackened aluminium disk than from an unblackened one does not prove that this heating effect may not be due to air, since the blackened surface may be imagined to *lay hold* of the air more than the metallic one. But the following sets of experiments prove that the heating effect of the aluminium disk with both sides blackened is the same as when only one side is blackened.

No. of set.	No. of observations in each set.	Time at full speed, in seconds.	No. of turns of handle at full speed.	Heat indication, in ° Fahr.	Tension of air.	
X.	2	30	20	0·9	1·1	Disk blackened on one side.
XIII.	3	30	20	0·8	0·4	Disk blackened on both sides.

It would therefore appear to be proved that in these experiments the heating effect is due to the increased temperature of the disk.

17. Before proceeding further it may be advisable to detail some experiments made with an ebonite disk $\frac{1}{10}$ inch thick. In these experiments care was taken that the ebonite should have the same temperature throughout its thickness, so that there might be no flow of heat from the interior to the surface, or *vice versâ*. The experiments were made with rock-salt in the cone.

No. of set.	No. of observations in each set.	Time at full speed, in seconds.	No. of turns of handle at full speed.	Heat indication, in divisions.	Nature of gas.	Tension of gas, in inches.	Probable purity of gas (absolutely pure gas=100).
XIV.	3	30	20	32	air	1·1	
XV.	1	30	20	30	air	0·26	
XVI.	1	30	20	31	air	1·1	
XVII.	2	30	20	28·5	air	0·26	
XVIII.	2	30	20	28·5	air	0·25	
XIX.	3	30	20	29	hydrogen	0·25	90

From these experiments it may be taken for granted that the heat indication given by an ebonite disk is, like that from an aluminium disk, independent both of the density and chemical constitution of the residual gas. It is also highly probable that the unknown cause of the heating effect is the same for both disks.

18. To return now to the aluminium disk, it may be shown that the heating of this disk is not caused by revolution under the earth's magnetic force; for (1) the following calculation, kindly furnished by Professor Maxwell, shows that the heating effect due to this cause would, for the aluminium disk $\frac{1}{20}$ of an inch in thickness, amount, under the circumstances of rotation, only to $\frac{1}{61270}$ of a degree Fahr., whereas the observed effect is more than half a degree.

An ellipsoid, semiaxes a, b, c, revolves about the axis c with velocity ω,

in a uniform magnetic field. To find its electrical state at a given instant. At the given instant let the axes of x, y, z coincide with a, b, c; then using the notation in the paper on the Electromagnetic field *,

$$\mathrm{P}=\mu\omega\gamma x-\frac{d\psi}{dx}=p\rho,\ \mathrm{Q}=\mu\omega\gamma y-\frac{d\psi}{dy}=q\rho,\mathrm{R}=\ -\mu\omega\,(\beta y+\alpha x)-\frac{d\psi}{dz}=r\rho,$$

P Q R electromotive force, $\alpha\,\beta\,\gamma$ magnetic intensity, ψ electric tension, μ=coefficient of magnetic induction=1 for everything but iron, p, q, r electric currents, ρ=resistance of cubic unit of volume.

The condition of the currents being confined to the ellipsoid, is

$$\frac{px}{a^2}+\frac{qy}{b^2}+\frac{rz}{c^2}=0.$$

Solving, we get

$$p=\frac{\mu\omega}{\rho}\ \frac{a^2\alpha}{a^2+c^2}z,\qquad q=\frac{\mu\omega}{\rho}\ \frac{b^2\beta}{b^2+c^2}z,\qquad r=-\frac{\mu\omega}{\rho}\left(\frac{\alpha x}{a^2+c^2}+\frac{\beta y}{b^2+c^2}\right)c^2,$$

$$\psi=\tfrac{1}{2}\mu\omega\gamma(x^2+y^2)-\mu\omega z\left(\frac{a^2\alpha x}{a^2+c^2}+\frac{b^2\beta y}{b^2+c^2}\right)+\mathrm{C}.$$

This is the complete solution. The heat (measured as energy) produced in unit of volume in unit of time is $\rho(p^2+q^2+r^2)$. The whole heat produced in the ellipsoid in unit of time is

$$\frac{4\pi}{15}\,\frac{\mu^2}{\rho}\,\omega^2\,abc^3\,\left\{\frac{a^2\alpha^2}{a^2+c^2}+\frac{\beta^2b^2}{b^2+c^2}\right\}.$$

If $a=b$, this is $$\frac{4\pi}{15}\,\frac{\mu^2}{\rho}\,\frac{a^4c^3}{a^2+c^2}\,(\alpha^2+\beta^2).$$

If c is small compared with a, it becomes

$$\frac{4\pi}{15}\,\frac{\mu^2}{\rho}\omega^2a^2c^3(\alpha^2+\beta^2).$$

If the axis is horizontal,

$$\alpha^2+\beta^2=\mathrm{H}^2\sin^2\theta+\mathrm{V}^2,$$

where H=horizontal magnetic force, and V=vertical magnetic force, and θ=angle between the axis of rotation and the magnetic meridian, a=radius and c half the thickness, $\omega=2\pi n$, where n denotes the revolutions per second; $\mu=1$; ρ is the resistance of unit length and unit section.

Now, the resistance of 1 metre long and 1 millimetre diameter

$$=\frac{4\rho}{\pi}10^6=0{\cdot}0375\times10^7$$

for aluminium in metrical units by Matthiessen, or $\frac{\pi}{\rho}=\frac{32}{3}$.

ρ is the same for metrical and for British measure.

At Kew horizontal force=3·81, dip 68°·10; $\theta=90°$ in the experiments $\therefore\ \alpha^2+\beta^2=104{\cdot}8$ British measure.

The revolving body is not an ellipsoid but a cylinder, equally thick throughout; to correct for this we shall put $\frac{15}{2}\,c^3$ for c^3.

We get for the energy converted into heat by electrical action per second 12·91 in grain-foot-second measure.

* Phil. Trans. 1865, p. 459.

Now $772g$=energy required to raise 1 grain of water 1° Fahr.

The disk $=\frac{70,000}{16}\times\cdot22$ grain of water, ∴ energy corresponding to 1°

$$=772\times\frac{32\cdot2}{16}\times15400,$$

or rise of temperature per second$=\frac{12\cdot91}{1540\times15400}=\frac{12\cdot91}{23716000}$ degree Fahr.

or about $\frac{1}{61270}$ degree in 30 seconds.

This is when the heat is uniformly distributed through the thickness of the disk, which it will be in less than thirty seconds. If there were no conduction, the rise of temperature at the surface would be about twice the value found above.

(2) It would appear from the above formula that the heating effect due to this cause should increase with the thickness of the disk. The following experiments show that, on the contrary, the heating effect, as regards temperature, diminishes as the thickness of the disk increases.

No. of set.	No. of observations in each set.	Time at full speed.	No. of turns of handle at full speed.	Heat indication, in ° Fahr.	Tension of air, in inches.	
XIII.	3	30	20	0·8	0·4	Aluminium disk $\frac{1}{20}$ inch thick.
XX.	3	30	21	1·7	0·4	Aluminium disk $\frac{1}{40}$ inch thick.

(3) The heat indication afforded by an ebonite disk is against the conclusion that this effect is due to rotation under the earth's magnetic force.

It would therefore appear to be proved that in these experiments the increased temperature of the aluminium disk is not due to rotation under the earth's magnetic force.

19. It might perhaps be said that the heating of the disk may be due to heat conducted from the bearings into the disk and then distributed outwards; and this conjecture will require to be examined, since the bearings are, no doubt, heated by friction during the motion. This heating effect on the bearings was measured by means of a very delicate thermometer, which was inserted into a small hole in the bush through which oil is supplied to the spindle, and made to be in metallic contact with the sides of this hole; the mean of three observations made the heating effect at the spindle due to rotation to be 4° Fahr.

In the next place, the aluminium disk, separated from its metallic spindle by the ebonite washer, and in every respect the same as when made to rotate, had its spindle heated artificially by a mercury bath, generally at least 40° Fahr. above the previous temperature. After the lapse of two minutes the effect upon the pile was hardly perceptible—not more than five divisions.

1) It would appear from this experiment that the heating effect ob-

served in rotation cannot be due to heat conducted from the bearings through the ebonite washer, since a temperature difference between the bearings and the disk ten times greater than that produced by rotation causes a heating effect at least six times less than that caused by rotation in a somewhat less time.

(2) The ebonite washer used to prevent the heat of the bearings from reaching the aluminium disk, is a cylindrical disk, its thickness being two-tenths of an inch, and the area of one of its faces 3·15 square inches. It is shielded behind by a brass disk, of similar size, which brass disk being near the bearings, and metallically connected with them, we may suppose to have the same temperature as the bearings. Thus one face of the ebonite washer is in contact with brass, having the temperature of the bearings, while the other is in contact with the aluminium disk. Supposing that this washer, used to protect the aluminium disk from the heat of the bearings, was of iron instead of being of ebonite, we can calculate approximately from Principal Forbes's determinations of the absolute conductivity of this metal, how much heat would be conducted across the washer during the experiment. According to these observations, if a cube of iron whose side is one foot, have one of its faces kept permanently at a temperature 1° C. higher than the opposite face, the quantity of heat conducted across in one minute will be ·011 unit nearly, a unit denoting the amount of heat required to raise a cubic foot of water 1° C. Since in these observations both the temperature difference and the unit are expressed in the same thermometric degrees, we may, if we choose, substitute degrees Fahr. for degrees Centigrade in the above expression for conductivity. Now, if we assume as an approximation that during the whole experiment of rotation the heat conducted across such a washer is the same as if for one minute the temperature difference between both sides of the washer were kept at 2° Fahr., and if we make allowance for the surface and for the thickness of the washer, we obtain the following expression as approximately representing the heat conducted across the washer during the experiment,

$$\text{Heat} = \cdot 011 \times 2 \times \frac{3 \cdot 15}{144} \times \frac{12}{\cdot 2} = \cdot 028 \text{ unit nearly,}$$

where the first factor is on account of the double temperature difference, the second on account of the surface, and the third on account of the thickness.

But a unit of heat in the above expression denotes the amount necessary to raise a cubic foot of water (or nearly 1000 ounces) 1° Fahr. Now the weight of the disk is 10·5 ounces, and its specific heat is 0·22. Hence the above amount of heat will raise the disk

$$\cdot 028 \times \frac{1000}{10 \cdot 5} \times \frac{1}{\cdot 22} = 12° \text{ Fahr. in temperature.}$$

Hence we see that if the material of the washer had been of the metal bismuth, of which the conductivity is 7 times less than that of iron, and if

we suppose the circumstances of the experiment to be equivalent to a temperature difference of 2° Fahr. between the two sides of the washer lasting for one minute, then the quantity of heat conducted across the washer will be a little greater than that observed. But the conductivity of ebonite is no doubt very much less than that of bismuth, and therefore on this account we cannot suppose that the heating effect observed is due to conduction.

(3) In this investigation no account has been taken of the unequal distribution of temperature from the centre to the circumference of the disk, the tendency of which would be to diminish the effect upon the pile (which was directed to the circumference of the disk) of the heat passing through the washer; and indeed, when this element is taken into account, it is not surprising to find, as was actually the case, that in some preliminary experiments, where the disk was metallically connected with the spindle, the effect was not greater than with the ebonite washer.

(4) The short time in which the effect attains its maximum value is against the supposition that it is caused by conduction from the bearings.

(5) The fact that (as we shall afterwards see) the temperature effect in three aluminium disks of different thicknesses is inversely proportional to the thickness, is also against this supposition.

(6) And so is the fact that a heat-effect obeying apparently the same laws, holds for an ebonite disk in which there is but a very feeble conduction.

On the whole, therefore, we cannot suppose this effect to be due to conduction, or at least we must conclude that the effect of conduction constitutes only an exceedingly small fraction of that observed.

20. It was suggested to the authors by Professor Stokes and by Mr. Grove, that the effect might be due to vibrations of the disk, the energy of which, owing to the viscosity of the disk for such vibrations, might ultimately become converted into heat; and it is necessary to examine this question.

(1) The thickest aluminium disk was found to be out of truth not more than ·015 inch on each side. Hence, the thickness of this disk being ·05 inch, when turned with moderate rapidity, its apparent thickness should be

$$\cdot 015 + \cdot 05 + \cdot 015 = \cdot 08\,;$$

and experiment showed that when turned very fast, its apparent thickness was no greater. The greatest possible range of vibrations of the disk at its circumference could not, therefore, be more than ·015 inch on either side of the position of rest.

Again, it was ascertained by means of the note given by this disk, that it vibrates about 250 times per second.

Let us suppose the whole mass to have the same range of excursion (this will of course increase the result), the equation of vibration (not allowing for loss by viscosity) is

$$x = \cdot 015 \cos nt$$

and also time of vibration

$$=\frac{2\pi}{n}=\frac{1}{250}.$$

Hence

$$n=500\times 3{\cdot}14=1570, \text{ say,}$$

$$\therefore \frac{dx}{dt}=-\overset{\text{in.}}{{\cdot}015}\times 1570 \sin nt, \therefore \text{ greatest velocity } =\overset{\text{in.}}{23{\cdot}55},$$

or say 2 feet per second.

Hence the energy of this motion in foot pounds,

$$=\text{weight of disk in pounds}\times\cdot\frac{v^2}{2g}$$

$$=\text{weight of disk in pounds through } \tfrac{1}{16} \text{ of a foot.}$$

But an approximate experiment performed by causing the disk to ring, and noticing how long the sound lasted, would seem to show that probably the energy of vibration of the disk diminishes at first, and therefore constantly (if it is maintained) at the rate of the whole in 3 seconds. Hence in 30 seconds it loses 10 times as much as the whole; that is to say, in 30 seconds the heat produced cannot be greater than that due to the energy produced by the disk falling under gravity through $\frac{10}{16}$ of a foot. Reducing this to its heat-equivalent, the greatest possible heat effect due to vibration during 30 seconds rapid turning will be less than

$$\frac{10}{16}\times\frac{1}{772}\times\frac{1}{{\cdot}22}=\frac{1}{272}{}^{\circ} \text{ Fahr.},$$

which is a very small fraction of the effect observed.

(2) The thin aluminium disk was out of truth about ·02 inch on each side. Its note of vibration was as nearly as possible one octave lower than that of the thick disk, while its coefficient of viscosity was somewhat greater, say in the proportion of 3 to 2, than that of the thick disk. On the supposition that the heat generated is due to vibration, if we call the heat generated during 30 seconds in the thick disk $=1$, then that generated during the same time in the thin disk ought to be

$$1\times\frac{1}{2^2}\times\left(\frac{{\cdot}02}{{\cdot}015}\right)^2\times\tfrac{1}{2}\times\tfrac{3}{2}=\tfrac{1}{3},$$

where the first factor is on account of difference of time of vibration, the second on account of difference of range, the third on account of difference of mass, and the fourth on account of difference of viscosity.

But the heating effect (as far as quantity of heat is concerned) produced in the thin disk is as nearly as possible the same as that produced in the thick disk.

This fact is therefore against the hypothesis that the heating effect is due to vibration.

(3) In order to estimate the effect (if any) of want of truth in the disk, the thick aluminium disk was purposely put out of truth about $3\frac{1}{2}$ times

its usual amount; but the heating effect was as nearly as possible the same in both cases, being 32 divisions of the scale in both.

On all these grounds it would appear that the heating effect cannot be due to vibration of the disk.

21. It is hardly necessary to mention that the heating effect cannot be due to radiation and convection from the wheelwork, which is no doubt slightly heated during the experiment, for the mass of this metallic matter is so great, that we cannot imagine it to be heated more than 1° Fahr. Now the radiation from this against the back of the disk may certainly be neglected, while the convection must be very small, since in the experiments the pressure of the air was very small. Besides, the heating effect, as will be seen shortly, was found to be independent of the pressure.

22. It has thus been shown that the disk is heated during the experiment, and that this heating effect—

(1) Is not due to rotation under the earth's magnetic force;

(2) Is not due to conduction of heat from the bearings;

(3) Nor to radiation or convection from the wheelwork;

(4) Nor to vibrations of the disk.

And in view of the large and constant nature of this heating effect it may be asserted that it cannot be sensibly due, either to one of these causes singly, or to their combined effect.

23. It will now be shown that the heating effect is independent both of the density and chemical constitution of the residual air and vapour around the disk.

In art. 16, if we compare together the 7th, 8th, 9th, 10th, 11th, and 12th sets of experiments, we shall see that the heating effect was sensibly the same, whether the residual gas was atmospheric air, or hydrogen or coal-gas.

As hydrogen diffuses very quickly, it might perhaps be supposed that when heated by rotation it might find access to the pile through the rock-salt cover more easily than heated atmospheric air, so that while the whole effect might appear the same in hydrogen as in air, yet only part of that in hydrogen might be due to radiant heat, the remainder being due to heated gas which had obtained access to the pile. This was, however, disproved by an experiment, which showed that by blackening the interior of the cone, the effect upon the pile was just as much diminished in a hydrogen vacuum as in an air vacuum; and hence in both cases the whole effect is due to radiant heat.

But, besides the residual gas, it may with truth be supposed that there is always more or less of aqueous vapour, and also a little of the vapour of oil, and perhaps of the vapour of mercury in the receiver. As regards the hygrometric state of the residual air and its influence on the disk, this would appear to be of the following nature:—

(1) When the vacuum has just been made, there is generally a hygrometric difference between the air and the surface of the disk, on account of

which there is a strictly temporary effect, either in the direction of heat or cold, at the surface of the disk, owing probably to condensation or evaporation of small quantities of aqueous vapour; but this effect disappears the moment the motion is stopped, leaving behind the permanent effect apparently unaltered.

(2) This temporary effect disappears when the disk has been left for some hours in the vacuum.

Next, with regard to vapour of oil, we cannot suppose its effect to be so large or so different in character and constancy from that of aqueous vapour as to account for the effect observed. Add to this that the effect takes place with an uncoated metallic disk probably to the same extent as with a coated one. The same remark may be made with regard to vapour of mercury. The effect would therefore appear to be independent of the chemical nature of the residual gas and vapour around the disk.

In order to prove that this effect is also independent of the pressure of the residual gas, it is only necessary to refer to the whole body of experiments which have been described, to see that between 4 inches and 0·25 inch there is no perceptible variation in the effect observed.

24. The following generalization may now be made:—

(1) If a perfectly true aluminium disk (without vibrations) be made to rotate in a vertical plane at the earth's surface, after the manner herein described, there will be an increase of the temperature of the disk, which is not due to communication of heat from the bearings or machinery, nor to the earth's magnetic force.

(2) This heating effect is independent of the density and chemical constitution of the residual air and vapour which surround the disk.

(3) It is probable that the quantity of heat developed in disks of similar extent of surface and similar circumstances of motion is the same. For, in the first place, the quantity of heat developed in three aluminium disks, ·05, ·0375, ·025 of an inch in thickness respectively, would appear to be the same, the relative thermometric effect for these disks varying inversely as their thickness, and being in the following proportions, 30, 43, 60, as determined by one complete set of experiments.

Again, the quantity of heat developed in the thick aluminium disk, with its surfaces both uncoated, was probably the same as when one surface was coated and one left bare, or as when both surfaces were coated.

25. The authors will not attempt here a further generalization, but they would desire to make one remark. In absence of definite knowledge of the nature of that medium which transmits radiant light and heat, it might be supposed possible that when a radiant body is in rapid motion, the intensity of its radiation is somewhat increased. But if we bear in mind that in these experiments the effect was observed after bringing the disk to rest, and that the temporary effect during rotation sometimes observed can probably be otherwise accounted for, we are forced to conclude that, as far as we may judge from these experiments

(and they are of a very delicate nature), there is no perceptible effect of motion upon radiation.

In conclusion the authors desire to say that they are much indebted to Mr. Beckley, who not only invented the apparatus, but assisted at all the experiments, and without whom they could not have been performed in a manner so satisfactory. They are also indebted to Mr. Atkinson for his kindness in lending them a large gasometer, and to Mr. Browning and Mr. Ladd for exceedingly true aluminium and ebonite disks.

III. "On the Bones of Birds at different Periods of their Growth." By John Davy, M.D., F.R.S., &c. Received October 23, 1866.

In this paper I beg to submit to the Royal Society the results of some further observations on the bones of birds, and more especially on those bones which in the adult contain air.

In offering them, I would wish them to be considered as a continuation of those communicated on a former occasion, and published in the Proceedings of the Society*.

In engaging in the inquiry I have had two objects chiefly in view: one to endeavour to determine whether at an early stage the bones, which at maturity contain air, differ essentially from those of birds which are then, and are permanently filled with marrow; another, to endeavour to ascertain in the instance of the former, the rate at which their early contents are absorbed, or an approximation to the time that air takes the place of the medulla.

The birds of the first kind subjected to observation have been the following: the common fowl, duck, goose, turkey, pheasant, partridge, grouse, rook, common crow, owl, sparrow-hawk, buzzard, blue tit; of the second kind, the woodcock, blackbird, water-ouzel, marten, swift, greenfinch, titlark, sparrow, stonechat, blackcap, yellow ammer, little sandpipe, canary.

I. *Of the Common Fowl.*—Of this bird I have examined the bones at different ages in a large number of instances. A few are selected as most characteristic :—

1. Of a fœtal chick taken from the egg on the fourteenth day of incubation, the bones of the extremities were much advanced, the inferior more than the superior. Their epiphyses were almost gelatinous; but the shafts were partially ossified, and had already some firmness. In both the humeri and femora, medullary matter was found. It was of a red colour, and under the microscope exhibited the usual character of this tissue. Broken up, in each were seen numerous oil-globules, with which were mixed blood-corpuscles and other corpuscles of a smaller size, some circular, some of an irregular form.

* Vol. xiv. 337, 440, 475.